

Displacement Current and the Generation of Parallel Electric Fields

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We show for the first time the dynamical relationship between the generation of magnetic field-aligned electric field (E_{\parallel}) and the temporal changes and spatial gradients of magnetic and velocity shears, and the plasma density in Earth's magnetosphere. We predict that the signatures of reconnection and auroral particle acceleration should have a correlation with low plasma density, and a localized voltage drop (V_{\parallel}) should often be associated with a localized magnetic stress concentration. Previous interpretations of the E_{\parallel} generation are mostly based on the generalized Ohm's law, causing serious confusion in understanding the nature of reconnection and auroral acceleration.

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Mechanisms for the acceleration of charged particles and for magnetic reconnection are two of the most important, long-standing questions in magnetospheric and cosmic plasma physics. The formation of E_{\parallel} is necessary and crucial for magnetic reconnection and for efficient acceleration of auroral particles [1–5]. Satellite observations in the auroral acceleration region and at current sheets in the magnetosphere in the last few decades have proven experimentally the existence of E_{\parallel} and perpendicular electric fields (\mathbf{E}_{\perp}), which often are structured as localized double layers, charge holes, or U-shaped voltage drops. It is known that if an electrostatic E_{\parallel} were to be established without supporting mechanisms, it would disappear quickly by the rapid rearrangement of the electrons and ions due to the electric force in order to restore plasma quasineutrality. How to establish and support E_{\parallel} in a collisionless plasma then becomes the important theoretical question. Most previous models of double layers, charge holes, U-shaped V_{\parallel} , and reconnection describe steady or quasisteady cases, which do not address this question. To date, the basic dynamical theory for the generation of electric fields, in particular, E_{\parallel} , has yet to be developed and the question is still unsolved.

Since ideal MHD theory excludes the existence of E_{\parallel} , it has been broadly accepted that the cause of E_{\parallel} can be obtained by analyzing the balance between the electric force and other terms in the parallel component of the generalized Ohm's law, or equivalently, the electron Newton's law. We show here that the force balance or $F = ma$ itself is unable to give the cause of the E_{\parallel} generation. Although this misunderstanding in the theory of the E_{\parallel} generation is subtle, it has impeded our understanding of the mechanisms of reconnection and auroral particle acceleration for several decades. We further derive the equations for the generation of E_{\parallel} and V_{\parallel} using a complete set of dynamical equations including the displacement current, and explore possible related physical processes causing $\partial E_{\parallel}/\partial t$ and $\partial V_{\parallel}/\partial t$. In order to show the basic physical considerations in this short Letter, our derivation will be carried out only for the simplest possible case.

We begin by examining the parallel component of the generalized Ohm's law, which is essentially Newton's law for the electrons, i.e.,

$$m_e \frac{du_{\parallel e}}{dt} \approx -eE_{\parallel} - m_e \nu^* u_{\parallel e} - \frac{(\nabla \cdot \mathbf{P}_e)_{\parallel}}{n}, \quad (1)$$

where n is the electron number density. The mean effective collision frequency ν^* comes from anomalous resistivity or other nonideal or kinetic effects. Equation (1) shows that the electron acceleration is caused by the electric force $F_{E_{\parallel}} = -eE_{\parallel}$, resistive force $F_{\nu^*} = -m_e \nu^* u_{\parallel e}$, and/or the "force" describing momentum transfer by the pressure gradient $F_{\nabla P} = -(\nabla \cdot \mathbf{P}_e)_{\parallel}/n$. Assuming that the ion parallel current is negligible, Eq. (1) can be rewritten in terms of $J_{\parallel} \approx -neu_{\parallel e}$,

$$E_{\parallel} \approx \frac{4\pi}{\omega_{pe}^2} \frac{dJ_{\parallel}}{dt} + \eta^* J_{\parallel} - \frac{(\nabla \cdot \mathbf{P}_e)_{\parallel}}{ne}, \quad (2)$$

where $\eta^* = 4\pi\nu^*/\omega_{pe}^2$, $dJ_{\parallel}/dt = \partial J_{\parallel}/\partial t + [\nabla \cdot (\mathbf{u}\mathbf{J} + \mathbf{J}\mathbf{u} - \mathbf{J}\mathbf{J}/n_e e)]_{\parallel}$, and $\omega_{pe} = (4\pi n e^2/m_e)^{1/2}$ is the plasma frequency. Analyses based on the electron Newton's law have often been used to explain reconnection onset or auroral particle acceleration. By emphasizing different terms on the right-hand side of Eq. (2), various models have been developed and accepted, such as the electron inertia model, the anomalous resistivity model, and models associated with $(\nabla \cdot \mathbf{P}_e)_{\parallel}$, to explain reconnection onset or auroral acceleration. Do these expressions provide a mechanism for the generation of E_{\parallel} ? In considering the first term on the right-hand side of Eq. (2), the expression $E_{\parallel} \approx (4\pi/\omega_{pe}^2)(dJ_{\parallel}/dt)$ states that an electric force causes the acceleration of an electron. Clearly, the acceleration term itself cannot be the cause of the E_{\parallel} generation and particle acceleration. For the resistive and pressure terms, the expressions $E_{\parallel} \approx \eta^* J_{\parallel}$ and $E_{\parallel} \approx -(\nabla \cdot \mathbf{P}_e)_{\parallel}/ne$ describe the force balance between electric force and resistive force or the pressure gradient force in the parallel direction, respectively. They do not imply causality. Indeed, if the forces are balanced, the electron acceleration term vanishes and the electrons cannot be accelerated.

Newton's law for electrons simply states that forces cause a change of the electron momentum. It does not give the cause of the forces or the relationship between the different forces. The causes of anomalous resistivity, the electron pressure gradient, and E_{\parallel} are often different, although the three forces may be related. Therefore, deducing the mechanisms of the E_{\parallel} generation from the force balance is incomplete and often misleading.

Some steady state models are based on the kinetic theory. For example, in studies of the auroral zone, the Knight [6] kinetic current-voltage relations and its improved versions [7,8] have been used to explain auroral acceleration. However, none of these models reveals what dynamical processes generate and hold up E_{\parallel} and V_{\parallel} .

Analysis of the energy supply also clarifies this point. The total energy dissipation in the E_{\parallel} region is

$$J'_{\parallel} E_{\parallel} \approx \frac{\partial(E_{\parallel}^2/8\pi)}{\partial t} + \frac{4\pi}{\omega_{pe}^2} J_{\parallel} \frac{dJ_{\parallel}}{dt} - \frac{J_{\parallel}(\nabla \cdot \mathbf{P}_e)_{\parallel}}{en_e} + \eta^* J_{\parallel}^2, \quad (3)$$

where $J'_{\parallel} = J_{\parallel} + (1/4\pi)\partial E_{\parallel}/\partial t$ is the total parallel current. The four terms on the right-hand side describe, respectively, the energy spent in changing the electric energy associated with E_{\parallel} , the acceleration of electrons, the energy transport in the E_{\parallel} region by building up or smoothing out the inhomogeneity of density and/or temperature, as well as Joule heating caused by anomalous resistivity. With a larger η^* or larger positive $-J_{\parallel}(\nabla \cdot \mathbf{P}_e)_{\parallel}/en_e$, less energy is directed toward generating a E_{\parallel} and producing a coherent electron acceleration. We will show later in this Letter that the storage and release of the free magnetic energy and the kinetic energy of plasma bulk flow are related to the local generation of E_{\parallel} and V_{\parallel} . However, in a steady or quasisteady model, the input and output energies are balanced on average and the electric field could be electrostatic. Then, the dynamical processes associated with the change of magnetic and kinetic energy accompanied with a curled electric field do not appear explicitly in the steady state case, and hence are often overlooked.

On the other hand, the relation between auroral acceleration and magnetic stress concentration has been noted by Haerendel [9] in his fracture model, although a detailed theory of the formation of the fracture has not been given. Schindler *et al.* [10] emphasize kinematically the relationship between (i) the magnetic field \mathbf{B} , the plasma bulk flow \mathbf{u} , or the "slippage" between \mathbf{B} and \mathbf{u} , and (ii) a parallel voltage drop V_{\parallel} . However, the dynamical processes associated with the generation of E_{\parallel} and V_{\parallel} have not been discovered simply due to a lack of a full dynamical treatment.

In order to find dynamical processes that generate and maintain E_{\parallel} and V_{\parallel} , we first derive the dynamical equations. The generation of the electric field is represented by the displacement current $(1/4\pi)\partial \mathbf{E}/\partial t$ [11], as shown in the parallel component of Ampere's law,

$$\frac{1}{4\pi} \frac{\partial E_{\parallel}}{\partial t} = \frac{c}{4\pi} (\nabla \times \mathbf{B})_{\parallel} - J_{\parallel}. \quad (4)$$

Although the magnitude of the parallel displacement current $J_{\parallel D} = (1/4\pi)\partial E_{\parallel}/\partial t$ is often very small relative to the current for most MHD phenomena, neglecting this term obscures the mechanism of the generation and the maintenance of E_{\parallel} . We define Γ as the ratio between $J_{\parallel D}$ and J_{\parallel} , i.e.,

$$\Gamma = \left| \frac{(1/4\pi)\partial E_{\parallel}/\partial t}{J_{\parallel}} \right| \approx \frac{\gamma_0^2}{\omega_{pe}^2} \propto \frac{\gamma_0^2}{n}, \quad (5)$$

where $E_{\parallel} \approx (4\pi/\omega_{pe}^2)(\partial J_{\parallel}/\partial t)$ is assumed and γ_0 is used to denote the time derivative. From Eq. (4), for a given $(\nabla \times \mathbf{B})_{\parallel}$, the generation of E_{\parallel} is more efficient if Γ is significant. It must be noted that even Γ is relatively small, it can give significant E_{\parallel} . For example, the establishment of $E_{\parallel} = 100$ mV/m in 1 s gives a displacement current of 10^{-6} $\mu\text{A}/\text{m}^2$, which is much smaller than typical auroral currents of ~ 1 $\mu\text{A}/\text{m}^2$. When the plasma density is low, and/or the changes in E_{\parallel} fast, γ_0^2/n becomes more significant and the displacement current becomes important relative to the current. The result follows intuitive sense. At low density, for a certain $(\nabla \times \mathbf{B})_{\parallel}$, the plasma is too tenuous to carry the J_{\parallel} and $\partial E_{\parallel}/\partial t$ becomes more significant. The estimate given by Eq. (5) should be treated as a lower limit since the presence of kinetic effects changes the parallel electric field. In addition, there is a positive feedback in the change of plasma density, since a localized E_{\parallel} accelerates particles away from the E_{\parallel} region, thus lowering the density and increasing the probability of an enhanced E_{\parallel} . The parameter Γ as defined by Eq. (5) can also be written as $\Gamma \approx |[\partial(E_{\parallel}^2/8\pi)/\partial t]/J_{\parallel}E_{\parallel}| \propto \gamma_0^2/n$, the ratio between the increased electric energy and the kinetic energy of the particles. In general, for a low plasma density case, more input energy can be used to generate a localized E_{\parallel} causing efficient accelerate of charged particles to high energy.

In order to give the basic physical picture of the E_{\parallel} generation, we discuss here the simplest possible case, without losing the main physical considerations. The auroral acceleration region can be approximately treated as a low β , cold plasma region, where $\beta = P_{\text{th}}/P_B$ and P_{th} and P_B are the plasma thermal pressure and magnetic pressure, respectively. We consider an auroral flux tube, where the background magnetic field $B_{\parallel}\hat{\mathbf{z}}$ is assumed strong. For current structures on the scale of auroral arcs, plasma compression can be assumed small, $\partial B_{\parallel}/\partial t \approx 0$ [12]. Since the background magnetic field is strong, the ideal Ohm's law applies in the perpendicular direction, so the electric field is $\mathbf{E} \approx -(1/c)\mathbf{u} \times \mathbf{B} + E_{\parallel}\hat{\mathbf{z}}$, where the perpendicular scale sizes are assumed to be larger than characteristic scale sizes such as the ion inertial length and $\eta^* \mathbf{J}_{\perp}$ is negligible. These assumptions must be modified for the current sheet case where the background magnetic field may be weak and plasma pressure gradient term should be included.

The relationship between (i) the temporal changes and spatial gradients of the total parallel current J'_{\parallel} and (ii) the

temporal changes and spatial gradients of the vorticity can be found from a full set of the equations. The J'_{\parallel} generation can be obtained from Eq. (4) and the curl of Faraday's law and Ampere's law,

$$\frac{\partial J'_{\parallel}}{\partial t} \approx -\frac{c^2}{4\pi} \nabla_{\parallel} (\nabla_{\perp} \cdot \mathbf{E}_{\perp}) = \frac{c}{4\pi} \nabla_{\parallel} (\mathbf{B} \cdot \boldsymbol{\Omega}) - \nabla_{\parallel} (\mathbf{u} \cdot \mathbf{J}'). \quad (6)$$

In the auroral zone case, the background magnetic field is large so the term involving the vorticity $\boldsymbol{\Omega} = \nabla \times \mathbf{u}$ dominates, and the term including $\mathbf{u} \cdot \mathbf{J}'$ is small. Then Eq. (6) can be approximated as

$$\left(1 + \frac{c^2}{V_A^2}\right) \frac{\partial \Omega_{\parallel}}{\partial t} - \frac{c^2}{V_A^2} [\nabla \times (\mathbf{u} \times \boldsymbol{\Omega})]_{\parallel} = \frac{4\pi c}{B} \left(\frac{\partial J'_{\parallel}}{\partial z} + \frac{(\mathbf{B}_{\perp} \cdot \nabla_{\perp}) J_{\parallel}}{B_{\parallel}} - \frac{(\mathbf{J} \cdot \nabla) B_{\parallel}}{B_{\parallel}} \right). \quad (9)$$

Considering only the dominant terms, substituting in the dielectric constant $\varepsilon = 1 + c^2/V_A^2$, and assuming $|(\mathbf{B}_{\perp} \cdot \nabla_{\perp}) J_{\parallel}| \ll |B_{\parallel} (\partial J_{\parallel} / \partial z)|$ and $|\mathbf{J} \cdot \nabla B_{\parallel} / B_{\parallel}| \ll |\partial J'_{\parallel} / \partial z|$, which is valid when the parallel wavelength of the wave is much less than the gradient scale length of the geomagnetic field, Eq. (9) becomes

$$\frac{\partial \Omega_{\parallel}}{\partial t} - \left(\frac{\varepsilon - 1}{\varepsilon} \right) [\nabla \times (\mathbf{u} \times \boldsymbol{\Omega})]_{\parallel} \approx \frac{4\pi c}{\varepsilon B} \frac{\partial J'_{\parallel}}{\partial z}. \quad (10)$$

Together, Eqs. (7) and (10) describe coupling between the parallel and perpendicular dynamics of the system. The temporal changes and spatial gradients of the total parallel current are related to spatial gradients and temporal changes of the vorticity, respectively.

Consider as an example the generation of E_{\parallel} or V_{\parallel} in a magnetic flux tube with a radius R and height h . The areas of the cross sections perpendicular and parallel to the axis of the flux tube are $\Delta S = \pi R^2$ and $\Delta S_{\phi} = hR$, respectively. We denote the integration through a surface area element ΔS of parallel current J_{\parallel} , total parallel current J'_{\parallel} , and parallel vorticity Ω_{\parallel} , as $\mathbb{J}_{\parallel} = \oint J_{\parallel} dS$, $\mathbb{J}'_{\parallel} = \oint J'_{\parallel} dS = \mathbb{J}_{\parallel} + (1/4\pi) \partial (\oint E_{\parallel} dS) / \partial t$, and $\mathbb{O}_{\parallel} = \oint \Omega_{\parallel} dS$, respectively. We then derive volume integrated forms of Eqs. (7) and (10) by integrating over ΔS and along the field line from $z = 0$ to h ,

$$\frac{\partial}{\partial t} \langle \mathbb{J}'_{\parallel} \rangle h \approx \frac{c B_{\parallel}}{4\pi} \mathbb{O}_{\parallel} |_{0}^h, \quad (11)$$

$$\frac{\partial}{\partial t} \langle \mathbb{O}_{\parallel} \rangle h \approx \frac{4\pi c}{\varepsilon B_{\parallel}} \mathbb{J}'_{\parallel} |_{0}^h. \quad (12)$$

Here, the volume integrals of J_{\parallel} and Ω_{\parallel} can be estimated to be $\int_0^h \mathbb{J}_{\parallel} dz = \langle \mathbb{J}_{\parallel} \rangle h$ and $\int_0^h \mathbb{O}_{\parallel} dz = \langle \mathbb{O}_{\parallel} \rangle h$. The volume integration of J'_{\parallel} is $\langle \mathbb{J}'_{\parallel} \rangle h = \langle \mathbb{J}_{\parallel} \rangle h - (\Delta S / 4\pi) (\partial V_{\parallel} / \partial t)$, where $V_{\parallel} \Delta S = -\int dS \int_0^h E_{\parallel} dz$.

In cylindrical coordinates, by definition, the azimuthal magnetic flux and velocity flux passing through ΔS_{ϕ} are $\Phi_B = \int_0^h dz \int_0^R dr B_{\phi}$ and $\Phi_u = \int_0^h dz \int_0^R dr u_{\phi}$, respec-

$$\frac{\partial J'_{\parallel}}{\partial t} \approx \frac{c B_{\parallel}}{4\pi} \frac{\partial \Omega_{\parallel}}{\partial z}. \quad (7)$$

The temporal change of vorticity is found by taking the curl of the MHD momentum equation,

$$\begin{aligned} \frac{\partial \boldsymbol{\Omega}}{\partial t} - \nabla \times (\mathbf{u} \times \boldsymbol{\Omega}) &= \nabla \times \left(\frac{1}{\rho c} \mathbf{j} \times \mathbf{B} \right) - \nabla \left(\frac{1}{\rho} \right) \times \nabla p \\ &\approx \nabla \times \left(\frac{1}{\rho c} \mathbf{j} \times \mathbf{B} \right). \end{aligned} \quad (8)$$

When the right-hand side of Eq. (8) is zero, the vorticity is frozen in to convection. The parallel component of Eq. (8) can be rewritten as

tively. We assume that J'_{\parallel} and Ω_{\parallel} are independent of r within the flux tube. Then the azimuthal magnetic field and velocity become $B_{\phi} = (2\pi r/c) J'_{\parallel}$ and $u_{\phi} = (r/2) \Omega_{\parallel}$, respectively. In this simplified case, the relations between (i) $\langle \mathbb{J}'_{\parallel} \rangle h$ and Φ_B and (ii) $\langle \mathbb{O}_{\parallel} \rangle h$ and Φ_u are

$$\langle \mathbb{J}'_{\parallel} \rangle h = \int_0^h dz \int_0^R J'_{\parallel} 2\pi r dr \approx c \int_0^h dz \int_0^R B_{\phi} dr \approx c \Phi_B, \quad (13)$$

$$\begin{aligned} \langle \mathbb{O}_{\parallel} \rangle h &= \int_0^h dz \int_0^R \Omega_{\parallel} 2\pi r dr \approx 4\pi \int_0^h dz \int_0^R u_{\phi} dr \\ &\approx 4\pi \Phi_u. \end{aligned} \quad (14)$$

Equation (13) can be rewritten as $\partial \langle V_{\parallel} \rangle_S / \partial t = \langle \mathbb{J}'_{\parallel} \rangle \times (4\pi h / \Delta S) - (4\pi c / \Delta S) \Phi_B$. The localized azimuthal magnetic flux is associated with the temporal change of voltage drop. From Eqs. (11) and (12), we obtain

$$c \frac{\partial \Phi_B}{\partial t} = \frac{c B_{\parallel}}{4\pi} \mathbb{O}_{\parallel} |_{0}^h, \quad (15)$$

$$\frac{\partial \Phi_u}{\partial t} = \frac{c}{\varepsilon B_{\parallel}} \mathbb{J}'_{\parallel} |_{0}^h. \quad (16)$$

Equation (15) [or (16)] shows that the azimuthal magnetic flux Φ_B (or velocity flux Φ_u) through ΔS_{ϕ} changes when the parallel vorticity \mathbb{O}_{\parallel} (or the parallel current \mathbb{J}'_{\parallel}) are different through a differential vertical velocity shear (or magnetic shear) at the two ends of the flux tube $z = 0$ and $z = h$. To understand the generation of E_{\parallel} or V_{\parallel} is to understand dynamical processes which cause the temporal changes of sheared magnetic flux Φ_B and velocity flux Φ_u , and spatial gradients of magnetic and velocity shears. In a steady or quasisteady model, the local change of magnetic field and magnetic flux does not appear in the relevant equations explicitly; therefore, the mechanism of the generation of E_{\parallel} or V_{\parallel} required by auroral particle acceleration and reconnection onset cannot be found from such models.

The ultimate energy source for most dynamical processes in the magnetosphere is the relative kinetic energy between relatively moving plasmas, which can be converted into the kinetic and magnetic energy carried by MHD waves. For example, in the auroral acceleration region, Alfvén waves carry both the free magnetic energy and kinetic energy of plasma bulk flow from the generator to the auroral E_{\parallel} region and the ionosphere. The equation of energy conversion in an auroral flux tube is approximately

$$\frac{\partial[(B_{\phi}^2 + E_{\perp}^2)/8\pi + \rho u_{\perp}^2/2]}{\partial t} + \nabla \cdot \mathbf{S} \approx -J_{\parallel} E_{\parallel} \approx -\left(\frac{\partial(E_{\parallel}^2/8\pi)}{\partial t} + \frac{4\pi J_{\parallel}}{\omega_{pe}^2} \frac{dJ_{\parallel}}{dt} - \frac{J_{\parallel}(\nabla \cdot \mathbf{P}_e)_{\parallel}}{en_e} + \eta^* J_{\parallel}^2\right), \quad (17)$$

where \mathbf{S} is the electromagnetic and kinetic energy flux through the flux tube.

Since a local azimuthal magnetic flux Φ_B is needed to support a localized V_{\parallel} , the existence of a localized dynamo effect ($\mathbf{J}_{\perp} \cdot \mathbf{E}_{\perp}^{\text{local}} < 0$), which is related to an increase of a localized Φ_B , becomes necessary. The release of the azimuthal magnetic flux Φ_B would support a localized shear flow (vortices), maintaining the parallel voltage drop. The time scale to release the Φ_B is the time scale over which the V_{\parallel} or auroral particle acceleration can be supported. It must be noted that for the $\mathbf{J}_{\perp} \cdot \mathbf{E}_{\perp}^{\text{local}} > 0$ case, the “load” process does not always mean an irreversible dissipation, but often implies the conversion of magnetic energy into the kinetic energy of plasma bulk flow, which can support a charge condensation accompanied by a E_{\parallel} .

From these equations, the generation of a sustained E_{\parallel} and V_{\parallel} is associated with a low plasma density as well as a local change of azimuthal magnetic and velocity flux, leading to its inductive and capacitive nature. The inductive and capacitive properties of active plasma regions have been emphasized by Alfvén [13].

In summary, we have noted in this Letter that previous studies of the E_{\parallel} generation based on analysis of the different terms in the generalized Ohm’s law yield only a force balance or $F = ma$, not the E_{\parallel} generation itself. These theoretical considerations have misled and hindered research on reconnection and auroral acceleration in the past few decades. Because of the lack of an accurate theoretical understanding of the E_{\parallel} generation, the physical processes causing and supporting a E_{\parallel} in the active plasma regions have been “either neglected or not yet discovered,” as pointed out by Fälthammar [4].

In contrast to previous studies, we emphasize that the E_{\parallel} generation is described by the parallel displacement current. We have shown through analysis of the whole set of dynamical equations that the generation of a sustained E_{\parallel} and V_{\parallel} is associated with the temporal changes and spatial gradients of magnetic and velocity shears, and favors a low plasma density. Our work represents the first explicit elucidation of the formation and maintenance of a significant E_{\parallel} and V_{\parallel} in the cosmic plasma, albeit in a simplified case.

Our derivations further suggest that reconnection and the acceleration of charged particles are the natural results of plasma dynamical interactions. Identification of the dynamical processes that cause a localized concentration and conversion of magnetic and mechanical stresses sup-

porting E_{\parallel} and V_{\parallel} , as well as the dynamics of the plasma density distribution, thus becomes key to understanding reconnection and charged particle energization.

Our preliminary results predict that the signatures of reconnection and auroral acceleration should often have a high correlation with low plasma density, and that a localized voltage drop in general must be also associated with a localized magnetic stress concentration. The localized reconnection releases the topological constraints and allows the rearrangement of magnetic and velocity stresses. This potentially giving rise to the E_{\parallel} generation, in particular, for a low density case, which in turn induces charged particle energization. Detailed models of 3D Alfvénic reconnection and the formation of double layers and charge holes in the auroral region based on the results presented here will be addressed in future separate papers. The addition of plasma kinetic effects [14] will yield more detailed predictions of observed quantities such as the electron distribution function and the efficiency of converting Alfvénic Poynting flux into precipitating electron particle flux.

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